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Incompressible Viscous Fluid Flow: Boundary Element Method

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Abstract:

The boundary element approach is used to develop an integral equation for the steady flow of a viscous fluid. The flow field is calculated using the continuity, Navier-Stokes, and energy equations. Differential equations regulating a given situation velocity-pressure-temperature is used to generate basic variables. The computation of the solutions that are essential Shown is the tensor. Straightforward flow scenarios are simple to apply to. Such as a driven cavity, step, deep cavity, and channel with several obstructions. Presented. There are indications of convergence issues, which have restricted the applications to which they may be used. Low Reynolds number flows are described here. Fundamental solution, boundary elements, integral equations

Introduction

It was evident from the first day that a set of partial differential equations (PDEs) was needed to simulate fluid flow through pipes, blade passageways, nozzles, and other channels. The flow of fluid was simulated. The challenges that arise in getting information closed solutions, even for simple processes, need the use of only with the use of creative tactics can you produce new ideas. Answers to the equations and various flow patterns Practical considerations were taken into account. A variety of computational methods have been used, including finite-state machines (FSMs). Adding volume and boundary element to a finite difference know who's who in the world in light of the discovery of new algorithms, each of these strategies changed over time as computers became quicker. All regions during the last several years.

FDMs have been around for a while now. Implemented as a solution for flow issues. To put it another way, recent decades; it was still crawling in the 1970s.Both serve as a foundation for business codes that deal with the problem of solving flow problems. Essentially each and every one of them. There has been a limit to how much work may be put into the application in the sense that any new numerical approach that is found CPU time and storage space are claimed to be reduced by the technique of solution the following are some of the prerequisites However, the boundary element approach has advanced. Depending on where it has been used. It's been around for a while now. Solid mechanics and materials science have seen the most progress. Inconsistencies in the sound system Wrobel 1984; Banerjee and Butterfield 1981) and the most sluggish in the literature mechanics of fluids. This work employs a didactical method. The technique used to apply boundary elements to fluid problems is likewise aimed towards educating newcomers to the approach. Input and output According to Kakuda and Tosaka (1988), implementation is based employs a new formulation of the boundary element approach in this case. Velocity components in the Navier Stokes equations for unsteady flow only a technique using the penalty function may be used used with great effectiveness in finite element flow analysis. TheIt was shown by a series of numerical examples. Examples. In 1985 and 1986, Tosaka and Onishi suggested new integrals. Steady-state and dynamic Navier-Stokes equations Problems with erratic flow. legitimacy and efficiency of the method Numerous numerical examples were used to demonstrate the methodology. Results for situations with a constant state of change Osaka and Kakuda (1986); Tosaka (1986)).

Although integral techniques were available several decades ago for \sthe application to flow issues of practical significance, a \comprehensive examination of the formulation and application to flow difficulties are still being examined more lately, since they are intended to ease considerably the storage and hopefully CPU time, Despite this apparent benefit, requiring less computing effort when volume integrals are turned into surface integrals, some drawbacks exist, such as increasing mathematical complexity in needed to acquire an useable computational formulation; the necessity for the computation of singular integrals; dense matrices whose inversion is more time expensive when contrasted with the banded matrices in the finite difference and finite element approaches. In the section Application below the application of the boundary element technique to the following fluid issues are shown: a) stepped channel, b) box with sliding lid, c) channel flow with numerous barriers, d) deep cavity flow e) channel flow with heat transfer

ρ = density
μ = dynamic viscosity
P = pressure
T = temperature
ν = absolute viscosity
k = thermal conductivity
c_v = specific heat at constant volume
c_p = specific heat at constant pressure

Situational Analysis

A domain in 2 R and its closed border, in, may be defined as the following:an ideal gas; the fluid's outer normal vector.(a, b) is a point of in R2 that is both incompressible and viscous. The consistentin cartesian coordinates, state conservation equations may be writtenas:

Conservation of Mass:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

x - direction

$$\mu\left[\nabla^2 u+\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\frac{\partial P}{\partial x}=\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) \qquad (2a)$$

y-direction

$$\mu\left[\nabla^2 v+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\frac{\partial P}{\partial y}=\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) \qquad (2b)$$

Conservation of Energy:

$$k\nabla^2 T=\rho c_v\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right] \qquad (3)$$

Let the following change of variables take effect in the conservation equations:

$$t^*=\frac{tV_\infty}{L} \quad P^*=\frac{P-P_\infty}{\rho V_\infty^2} \quad \rho^*=\frac{\rho}{\rho_\infty} \quad y^*=\frac{y}{L}$$

$$u^*=\frac{u}{V_\infty} \quad v^*=\frac{v}{V_\infty} \quad V_\infty=\sqrt{u_\infty^2+v_\infty^2} \quad \mu^*=\frac{\mu}{\mu_\infty} \quad \nu^*=\frac{\nu}{\nu_\infty}$$

For example, the Reynolds and Prandtl numbers are equal to Re and Prandtl, respectively.As a result, the equations for energy conservation are:

$$\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}=0 \tag{4}$$

Conservation of Momentum

x-direction

$$\nabla^2 u^* + \frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}\right) - \mathrm{Re}\frac{\partial P^*}{\partial x^*} = \mathrm{Re}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right) \tag{5a}$$

y-direction

$$\nabla^2 v^* + \frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}\right) - \mathrm{Re}\frac{\partial P^*}{\partial y^*} = \mathrm{Re}\left(u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right) \tag{5b}$$

Conservation of Energy

$$-\frac{1}{\mathrm{Re}}\nabla^2 T^* = \mathrm{Pr}\left(u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T}{\partial y^*}\right) \tag{6}$$

where [L] is a linear partial differential operator, {U}= {} T u v T P is the vector of the unknowns and {B} the vector of nonlinear convicted terms. Depending on the assumptions made, [L] and {B} can take different forms. For instance, vector {B} can be linearised and the linear terms included in [L]. Let, for the moment, all non-linear terms are included into {B}. Then

$$L_{IJ} = \begin{bmatrix} \nabla^2 + D_1D_1 & D_1D_2 & 0 & -\mathrm{Re}\,D_1 \\ D_2D_1 & \nabla^2 + D_2D_2 & 0 & -\mathrm{Re}\,D_2 \\ 0 & 0 & \frac{1}{\mathrm{Re}}\nabla^2 & 0 \\ D_1 & D_2 & 0 & 0 \end{bmatrix};$$

$$U_J = \begin{Bmatrix} u \\ v \\ T \\ P \end{Bmatrix};\quad B_I = \begin{bmatrix} \mathrm{Re}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) \\ \mathrm{Re}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) \\ \mathrm{Pr}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right) \\ 0 \end{bmatrix}. \tag{8}$$

(I, J=1, 2, 3, 4)

ere:

$$D_1 = \frac{\partial}{\partial x};\quad D_2 = \frac{\partial}{\partial y};\quad \nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{9}$$

If we use as a weight function, we may be able to come up with a solution. The adjoint of LIJ uses WIK as its basic solution tensor, as we'll see in a moment.an approach based on Hörmander's (1965) (Banerjee, 1994)weight function calculationA tensor and fundamental of WIK aresolution. Despite the fact that it does not directly give WIK, it does enableCombining many partial differential operators is the first stageWIK tensor from a single differential operator, LIJIs well planned. The basic solution or weight tensor WIKa solution to the steady Stokes heat conundrum may be foundtransfer:

$$\overline{L}_{IJ}W_{JK} + \delta_{IK}\delta(x-y) = 0 \tag{11}$$

where δ (x − y) is the Dirac delta function and LIJ is the adjoint operator of LIJ. Hörmander’s method is simultaneously applied to the continuity, Navier Stokes and energy equations for steady, incompressible flow:

$$[W] = -[L]^{-1}\delta(x-y) = -[I]\delta(x-y) \tag{13}$$

where, $[L]^{-1} = \mathrm{Adj}[L]/\mathrm{Det}[L]$ and $\mathrm{Adj}[L] = \mathrm{Cof}^{\mathrm{T}}[L]$

$$\mathrm{Cof}^{\mathrm{T}}[L] = \begin{bmatrix} D_2^2\nabla^2 & -D_2D_1\nabla^2 & 0 & -D_1\nabla^2\nabla^2 \\ -D_1D_2\nabla^2 & D_1^2\nabla^2 & 0 & -D_2\nabla^2\nabla^2 \\ 0 & 0 & \mathrm{Re}\nabla^2\nabla^2 & 0 \\ \frac{1}{\mathrm{Re}}D_1\nabla^2\nabla^2 & \frac{1}{\mathrm{Re}}D_2\nabla^2\nabla^2 & 0 & \frac{2}{\mathrm{Re}}\nabla^2\nabla^2\nabla^2 \end{bmatrix} \tag{14}$$

whose terms are $x_{ij} = (-1)^{(i+j)} m_{ij}$; m_{ij} are the minors of $[L]$, and

$$\det[L] = \nabla^2\left(\nabla^2\left(\nabla^2\right)\right) \tag{15}$$

Thus,

$$[W] = [L]^{-1}\delta(x-y) = \mathrm{Cof}^{\mathrm{T}}[L]\,(\det L)^{-1}[I]\delta(x-y) \tag{16}$$

Let

$$\phi^* = (\det[L])^{-1}\delta(x-y) \tag{17}$$

then

$$\det[L]\,\phi^* = \delta(x-y) \tag{18}$$

whose solution ϕ^* is:

$$\phi^*(x,y) = \frac{1}{128\pi} r^4 \ln r \tag{19}$$

The tensor WJK calculation is only calculated analytically, thus it is vital to be attentive in the deriving of these formulas.DiscretizationFormula (10) may be solved using Green-Gauss theorem.integral over the domain is turned into an integral over the domain integralseparated into ne by the contourboundaries are the next step.

$$\int_\Omega \left(L_{IJ}U_J - B_I\right)W_{JK}\,d\Omega = 0 \quad (I, J, K{=}1, 2, 3, 4) \tag{21}$$

in which the values of the variables at each boundary element are included, indicating that the differential equations have been converted into an algebraic equations system (21). If a person discovers thevalues of the variables at the border elements, the solutionThen, the border may be found.Green-Gauss theorem and the Green-Gauss formulaby multiplying each portion by a factor of, one gets the following equation:For any border element, this holds true:

$$
\begin{aligned}
C_{KI}(y)U_I(y) = &\int_\Gamma \{u_i(x)\Sigma_{IK}(x,y) - \tau_i(x,y)W_{IK}\} d\Gamma(x) + \\
&+\int_\Gamma \frac{1}{\mathrm{Re}}\left\{T(x)\frac{\partial W_{3K}}{\partial n(x)}(x,y) - q(x)W_{3K}(x,y)\right\} d\Gamma(x) \quad (22) \\
&+\int_\Omega B_I(x)W_{IK}(x,y)\,d\Omega(x)
\end{aligned}
$$

where recurring indices imply summing.

To emphasise the fact that equation (22)'s right hand side contains integrals both over the boundary and across the domain is necessary.the non-linear convective factors make BI difficult to modelcoefficient of the tensor, CIK, in equation (22).The boundary's geometry. It may have a value of 12, 1, or 0 if the pointWithin or outside the domain, y crosses a locally regular border.The border, as it were. If y is located at the extremum of, thenThere are two sides to every triangle: the tangents on each side of the centre line.also,to.

$$
q(x) = \frac{\partial T(x)}{\partial n} \qquad (23)
$$

$$
\Sigma_{iK}(x,y) = (-W_{4K}\delta_{ij} + W_{iK,j} + W_{jK,i})n_j \qquad (24)
$$

$$
\tau_i(x) = (-\mathrm{Re}\, p\delta_{ij} + u_{i,j} + u_{j,i})n_j \qquad (25)
$$

According to equation (37), it's important to note that some of the boundary conditions have predetermined values. Consequently, it is easy to use.shift things about, starting with the things you don't know.

as a result of the mandated values,

$$
\begin{aligned}
\delta &= [\delta_u \quad \delta_p]^T \\
\tau &= [\tau_u \quad \tau_p]^T
\end{aligned} \qquad (38)
$$

Rearrangement of matrices $[\overline{H}]$, $[G]$ and $\{D\}$ accordingly, results in

$$
\begin{bmatrix} \overline{H}_{uu} & \overline{H}_{up} \\ \overline{H}_{pu} & \overline{H}_{pp} \end{bmatrix} \begin{bmatrix} \delta_u \\ \delta_p \end{bmatrix} = \begin{bmatrix} \overline{G}_{uu} & \overline{G}_{up} \\ \overline{G}_{pu} & \overline{G}_{pp} \end{bmatrix} \begin{bmatrix} \tau_u \\ \tau_p \end{bmatrix} + \begin{bmatrix} D_u \\ D_p \end{bmatrix} \qquad (39)
$$

$$\Sigma_{21}=\frac{1}{\pi}\left\{\left[\frac{(x-x_i)^2(y-y_i)}{r^4}\right]n_1+\left[\frac{(x-x_i)(y-y_i)^2}{r^4}\right]n_2\right\}$$

$$\Sigma_{21}=\frac{1}{\pi}\left\{\left[\frac{(x-x_i)^2(y-y_i)}{r^4}\right]n_1+\left[\frac{(x-x_i)(y-y_i)^2}{r^4}\right]n_2\right\}$$

$$\Sigma_{22}=\frac{1}{\pi}\left\{\left[\frac{(x-x_i)\ (y-y_i)^2}{r^4}\right]n_1+\left[\frac{(y-y_i)^3}{r^4}\right]n_2\right\}$$

$$\Sigma_{11}=\frac{1}{\pi}\left\{\left[\frac{(x-x_i)^3}{r^4}\right]n_1+\left[\frac{(x-x_i)^2(y-y_i)}{r^4}\right]n_2\right\}$$

$$\Sigma_{24}=\frac{1}{\mathrm{Re}\,\pi}\left\{\left[-\frac{-2(x-x_i)(y-y_i)}{r^4}\right]n_1+\left[\frac{(x-x_i)^2}{r^4}-\frac{(y-y_i)}{r^4}\right]n_2\right\}$$

$$\Sigma_{14}=\frac{1}{\mathrm{Re}\,\pi}\left\{\left[-\frac{(x-x_i)^2}{r^4}+\frac{(y-y_i)^2}{r^4}\right]n_1+\left[\frac{-2\,(x-x_i)(y-y_i)}{r^4}\right]n_2\right\} \quad (26)$$

For constant boundary element, one has

$$\begin{cases} u_i(\Gamma_e)=u_{i_e} \\ \tau_i(\Gamma_e)=\tau_{i_e} \\ T(\Gamma_e)=T_e \\ q(\Gamma_e)=q_e \end{cases} \quad (27)$$

Substituting the indicated expressions into equation (22):

$$\begin{aligned} C_{KI}(y)U_I(y) = & \sum_{e=1}^{n_2}\int_\Gamma\left\{\Sigma_{iK}(x,y)u_{i_e}(x)-W_{iK}(x,y)\tau_i(x)\right\}d\Gamma(x)_e + \\ & +\sum_{e=1}^{n_2}\int_\Gamma\frac{1}{\mathrm{Re}}\left\{\frac{\partial W_{3K}(x,y)}{\partial n}T_e-W_{3K}(x,y)q_e\right\}d\Gamma_e + \\ & +\int_\Omega B_I(x)W_{Ik}(x,y)d\Omega(x) \end{aligned} \quad (28)$$

The Boundary Element Method Applied to Incompressible Viscous Fluid Flow J. of the Braz. Soc. of Mech. Sci. & Eng. Copyright 2005 by ABCM October-December 2005, Vol. XXVII, No. 4 / 459 where coma (,) stands for derivative with respect to the following index and summation is implied by repeating indices. From equation (24) the values of ∑IK are calculated:

Application

Five problems were selected to demonstrate the method's applicability: movement of a lid that causes recirculation in a square hollow at a constant speed) the flow is advancing in a new direction; and(c) the flow through an extremely deep hole flows across deep cavities with an elevated top surfacetemperature; e) the movement of water via a narrow canal dotted with impediments. The flow in the box is represented using a driven cavity. As depicted in Fig. 1, they are called "streamlines". The non-moving surfaces of the box have zero noslip, which is the boundary condition. And the moving slid's upper-surface velocity. Constant The temperature had been placed on the edge of the acceptable range. Fig. 1a's grid is 40x40 and has a resolution of 240px.to be used in Fig. 1b has dimensions of 30x30. As a result of this, the criteria for convergence were established. Discrepancy between what was predicted and what was actually found pressure, temperature, and velocity all go hand in hand. Convergence had been attained by this point in time the 400th instance of Reynolds' law. The bottom-right corner of the cavity shows evidence of recirculation. In a forward-moving stepped channel, the flow is simplified. Fig. 2a shows the facing step. To establish a boundary, the following criteria must be met: parabolic no-slip conditions on the walls and no dispersion of velocities at inflow The temperature of the wall is always the same. There is a grid of results given here.26×30. The anticipated point of reattachment is in line with previous predictions. Approaches such as the finite volume methods (Fig.In this section, (Rocamora F., 2002

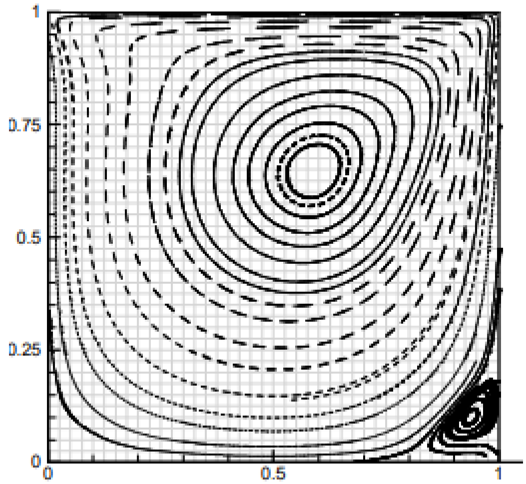


Figure 1a. Streamlines in the driven cavity flow Re = 300, grid 40×40, ε=0.0001

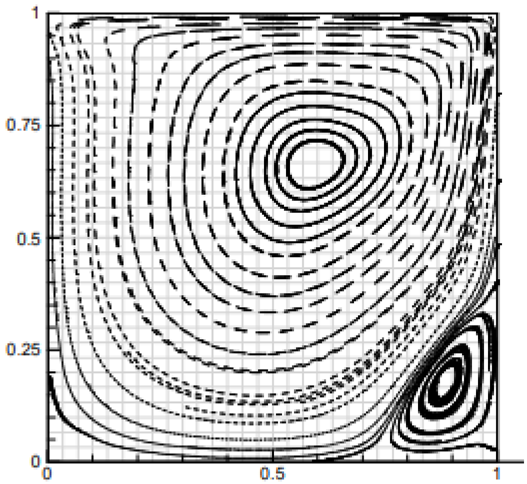


Figure 1b. Streamlines in the driven cavity flow Re = 400, grid 30×30, ε=0.001.

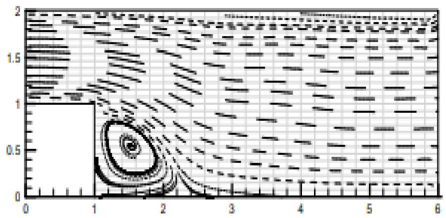
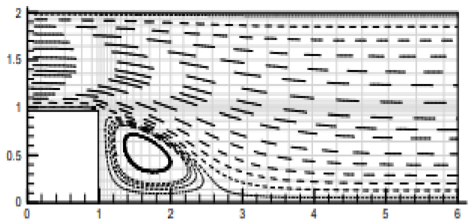


Figure 2a. BEM. Streamlines in the channel flow Re = 30, grid 26×30, ∈=0.0001



Conclusion

It has been shown that the boundary element approach may be used to calculate incompressible viscous flows. Despite the use of narrow grids, the results are excellent, catching Reverse flow areas may be found here. A comparison of the BEM's CPU use. Furthermore, the results for FVM show that BEM computations are much quicker. Ita variety of different uses, such as the aforementioned blade passages, which is the ultimate purpose of the study is being carried out. Blade passage flow necessitates a convergent technique. For a substantially higher Reynolds number. Using this approach, linearization of convective terms was used to facilitate the development infrastructural information that includes convection solution and so attain convergence at higher Reynolds numbers numbers.

References

Banerjee, P.K., 1994, "The Boundary Element Methods in Engineering", McGraw-Hill book Company Europe.
Banerjee, P.K., Butterfield, R. 1981 "Boundary Element Methods in Engineering Science", McGraw-Hill; New York.
Brebbia C. A., Telles T. C. F., Wrobel, L.C., 1984, "Boundary Element Techniques. Berlin. Heidelberg, New York: Springer.
Brebbia, C. A., Walker. S., 1980, Boundary Element Techniques in Engineering. Newnes-Butterworths. Hörmander, L., 1965, "Linear Partial Differential Operators", SpringerVerlag, Berlin.
Kakuda, K., Tosaka, N., 1988, "The Generalized Boundary Element Approach for Viscous Fluid Flow Problems", Boundary Element Methods in Applied Mechanics, Editors: M. Tanaka & T. A. Cruse, Pergamon Press, pp 305-314.
Partridge P.W, Brebbia, C.A., Wrobell L. C., 1992, "The Dual Reciprocity Boundary Element Method", Computational Mechanics Publications, Southampton.
Ramirez R.G. C., Barbosa J. R., 2004, "The Boundary Elements Method Applied to Incompressible Viscous Fluid Flow Problems", Anais em CD, III Congresso Nacional de Engenharia Mecânica, Belem – Para. Rocamora, Jr F. D, De Lemos M J S,.
"Heat Transfer Characteristic of Laminar and Turbulent Flows Past a Back Step in a Channel with a Porous Insert", 9 th Brazilian Congress of Thermal Engineering and Sciences, Caxambu-Minas Gerais, Brasil, 2002. Santos, Maria de Fatima de Castro Lacaz.,1998, "O Método de Elementos de Contorno Aplicado a Problems de Escoamento de Fluidos".
Tese Doutorado (Engenharia Aeronáutica e Mecânica) – Instituto Tecnológico da Aeronáutica, (Orientador) J. R. Barbosa.
Telles, J.C.F.A., 1987, "A Self-Adaptive Co-Ordinate Trans-formation for Efficient Numerical Evaluation of General Boundary Element Integrals, Int.
J. Numeric Methods in Engineering, 24: pp 956-973.
Tosaka, N. Kakuda, K. & Onishi, K., 1985, "Boundary Element Analysis of Steady Viscous Flows Based On P-V-U Formulation", Proc. 7nth Conference of BEM in Engineering, Lake Como, 71-80(9), Italy.
Tosaka, N., 1986, "Numerical Methods for Viscous Flow Problems Using an Integral Equation . In: Wang, S.Y., Shen, H.W., Ding, L. Z. (eds): River sedimentation, pp. 1514-1525: The University of Mississippi. Tosaka, N., Kakuda, K. 1986, "Numerical Solution of Steady incompressible Viscous Flow Problems by the Integral Equation Method".
In: Shaw. R. P., Periaux, J., Chaudouet, A., Wu, J., Marino, C., Brebbia, C.A. (eds): Innovative Numerical Method in Engineering, pp. 211-222. Berlin, Heidelberg, New York: Springer.
Tosaka, N., Onishi, K., 1985, "Boundary Integral Equation Formulation for Steady Navier Stokes Equations using the Stokes fundamental Solutions".
Eng. Analysis 2, 128-132. Tosaka, N., Onishi, K., 1986, "Integral Equation Method for Thermal Fluid Problems". In: Yagawa. G., Atluri, S. N. (eds): Computational Mechanics 86. vol. 2, pp. XI-103-XI-108. Berlin, Heidelberg, New York: Springer.

